CHANCE AND PROBABILITY

11

Missed the train, and why?

There is a small station named Bankhedi on the Itarsi-Allahabad railway line in the eastern part of Hoshangabad district. There is one train for Itarsi, Bina Express, which leaves Bankhedi at about 9:00 in the morning. Bina Express is usually 15 to 20 minutes late. But sometimes it comes exactly on time, and once or twice in a month it is even one or two hours late.

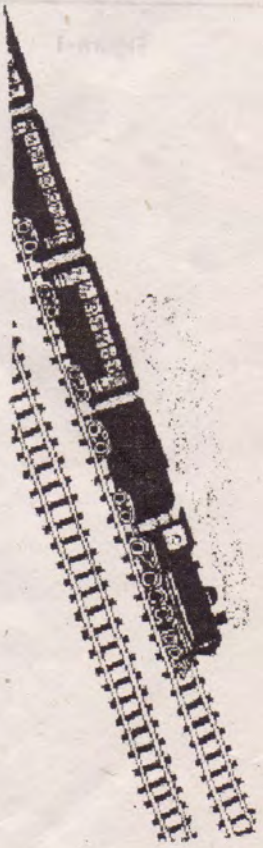
There is one other train, the Allahabad-Itarsi Passenger, that leaves Bankhedi for Itarsi at about quarter past two in the afternoon. But the Passenger rarely follows the Railway Time Table. Actually many times in a month the Passenger is 2-4 hours late. Maybe a couple of times each month the Passenger is less than 30 minutes late. Once in a while, maybe once in 2-4 months, the Passenger surprises everyone by coming at the right time. Now we shall ask you some questions in connection with this train situation. Discuss them among yourselves and answer. One gentleman reached Bankhedi station one hour late (about 10:00 in the morning) to catch Bina Express.

Do you think he would get the train or not? Give a reason for your answer. (1)

One day, another gentleman had to catch the Allahabad-Itarsi Passenger. He thought that since the Passenger is usually at least one hour late he would reach the station at about 3:00 in the afternoon (about 45 minutes late). Upon reaching the station he found that the train had left at the right time that day.

He really cursed his luck for missing the train.

Was the Passenger's coming at the right time an improbable event? In view of the history of the Passenger given above, give an answer with reasons. (2)



How many brinjals will there be? A matter of chance

A farmer who had brinjals growing on his farm had an average of 8 brinjals on each plant at any given time.

If you saw one of these plants before it started to bloom, could you correctly predict how many brinjals it would bear? Give a good reason for your answer. (3)

A game with cowries and *chiye*

You must have played games with tamarind seeds (*chiye*), cowries, dice and coins.

Figure-1

Have you ever been able to find a way to get chiye, cowries, dice, or coins fall the way you would prefer? (4)

Have you ever found one person always winning, or one person always losing? If not, what does happen? (5)

Suppose you throw 4 cowries 20 times - could it happen that the cowries land the same way every time? (6)

Can you always win or lose according to your desire? Give a good reason for your answer. (7)

Come, we will do some experiments and play some games in order to understand events like catching or missing a train, brinjals flowering or not flowering, and winning or losing dice games.

Heads or tails

Take a coin and flick it up so that it rapidly turns over and over before falling on the ground (figure 1). If it falls with the Ashoka Stambh on the top, it is heads; and if its the side with the number, it is tails.

Experiment 1

Heads or tails race - a game

All the students will play this game together. There should be at least 20 students for this game. If there are fewer than 20 students in your class, get some more students from classes 6 and 7. Bear in mind that they should also understand the game and make their own charts. To get ready for the game all should get together on the playground and draw 15 parallel lines on the ground, at a distance of one step from each other. All the lines should be long enough so that all the students can stand on them at the same time while being slightly separated from each other. Name the middle line, "0-Line" (Zero line). Counting out from the middle line, name the lines on one side, "Forward 1" "Forward 2", 'Forward 3", ... and on the other side, "Back 1", "Back 2", "Back 3", ... etc. Make your teacher the referee of the game.

The way to play the game

In the beginning all the students should stand on "0-Line" facing "Forward 1". Each student should hold one coin. When the referee blows the whistle each student should toss the coin and see if he/she gets a head or a tail. All who get heads should take one step forward to the "Forward 1" line, and the ones getting tails should take one step back to the "Back 1" line. This is the first move. In the following moves whoever gets heads should move one step forward and whoever gets tails should move one step back, from whatever line he/she is standing.

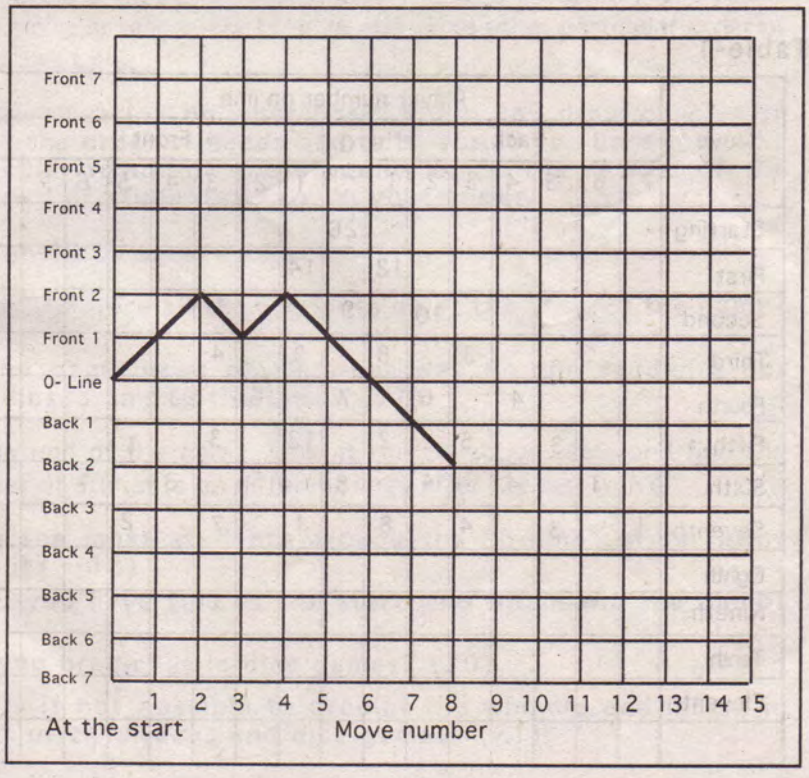
Every time the referee blows the whistle, all students should toss their coins and then make their moves. The first student to reach the "Forward 7" or "Back 7" lines is the winner of this race and with this the race ends.

Game diary

Two kinds of accounts of the game shall be kept.

1. Individual charts

Each student should record the details of his or her own moves in the "heads or tails race chart" given in the kit copy. An example of how to fill in the chart is explained here.



Before starting the game one student showed her position by making a large dot on the chart at the intersection of the "0-Line" and the vertical line "move number zero" indicating the position at the start. Now suppose the step by step moves of this student were: heads, heads, tails, heads, tails, tails, tails, tails. After making each move she made a large dot on the chart at the intersection of the line she had just reached and the line showing the number of the move (figure 2). After the game was finished she connected all the dots with straight lines. You should also make a chart like this to keep track of your moves. To do this, keep your kit copy in your hand while playing the game. After each move mark your position with a dot on the chart.

2. Collective Table

Your teacher (the game referee) will keep a chart of all the students in the game. The method of keeping the collective record is shown in Table- 1. In this sample table, the number of students on each line after a move in a sample game are shown. Obviously, the record of your game will be different.

Before starting the game, referee makes a blank table like this on the blackboard. To show the position at the "start" the number of students standing on "Line 0" is noted in the table. After each move, referee counts the number of students on each line and writes it down in the table. Maintain this record till the end of the game.

Table-1

Move	Player number on line															
	Back							0 Line	Front							
	7	6	5	4	3	2	1		1	2	3	4	5	6	7	
Starting								26								
First							12		14							
Second						10		9		7						
Third					5		8		9		4					
Fourth				4		6		7		6		3				
Fifth			3		5		2		12		3		1			
Sixth		1		4		4		8		5		3		1		
Seventh	1		3		4		8		1		7		2			
Eighth																
Nineth																
Tenth																
Elevanth																

Discussion on the game

After playing the game according to the above method, make your own individual chart.

Who won the race? (8)

Copy the collective table from the blackboard on a 'square' paper given in the kit copy. (10)

Look at your individual chart and tell if there was a particular order in your forward and backward moves during the game? (10)

All students should stick their individual charts on classroom walls. Look carefully at all the charts.

Was there any particular order in everyone's forward and backward moves? (11)

What would you call this kind of movement? (12)

Show the following on your individual chart, using a coloured pencil or red ink :

If you get heads every time you toss the coin what would your individual chart look like? (13)

If you get a tail after every head, and a head after every tail, what would your individual chart look like? (14)

It is possible that one or two students in a class may get heads every time, or tails every time, or get some other particular order of heads and tails.

In this situation on what basis would you draw conclusion about the order of heads and tails. Would you base it on the particular result of one student, or on the results of the majority of students? Explain your answer. (15)

Look at the collective table.

Which line were all the students on at the start of the game? (16)

As the game goes on, in what way do the students get distributed among the lines? (17)

At the end of the game, look at the collective table and find the number of students on different lines after the last move .

Were the most students around the "0-Line" or far away from it? (18)

Could you have told at the start who would win the game? (19)

Can you predict it in dice games? (20)

Why is it not possible to predict the winners and losers in heads or tails races and dice games? (21)

Another way of doing experiment 1 using blocks instead of coins

In case it is raining, you can also play the heads or tails race indoors. In place of coins, you can use the 1 cm^3 plastic blocks given in the kit. On any three sides of these cubes make a dot each. If you can not find colour for making the dots, stick a tiny bit of paper. Throw the cube to make each move. If the cube lands with a dot on the top, its heads, and tails otherwise.

Play the game in the manner previously explained. Take one chart for the heads or tails race from your kit copy. You know you are on "0-line" at the start of the experiment.

Put a dot on the chart to show your place at the "start" of the experiment. When the teacher (the referee) gives the signal, throw the cubes for the first move. But now you do not actually move. You just put a dot on your individual chart, forward or back from "0-Line", according to whether you got heads or tails. You move like this every time the referee gives the signal and keep a record of your moves on the chart. As soon as your dot reaches "Forward 7" or "Back 7" stand up and let the class know. As in the last game, the experiment is over once somebody reaches "Forward 7" or "Back 7". Just like that game, the referee should make a collective chart on the blackboard in this experiment also. After each move the referee will find out how many students have reached each line by asking all the students on a line to raise their hands. Based on the results of this experiment, write the answer to questions 8 to 21 in your note book.

A brain teaser

Could all the students win the heads and tails race together? Give a good reason for your answer. (22)

What will happen if the race is continued, even after a student reaches the seventh line? Think about it before you answer. (23)

Heads more, or tails

Which line a player would reach at the end of the experiment depends on whether he/she gets more heads or more tails. Whoever ends up on "0-Line" must have got an equal number of heads and tails. The players who end up on "Forward 2" got 2 more heads than tails and those on "Back 2" got 2 more tails than heads. Whoever got more tails will be that much further back from the "0-line".

After the last move, what is the difference between the number of heads and tails of most players? (24)

Two Main questions

Two main questions arise after the game is over:

Did you get a tail after each head and a head after each tail?
If not, did you get an equal number of heads and tails?
If this was not the case either, then what did happen?

May be you found some glimpses of an answer to these questions from the game. The answers can only be confirmed by repeating the experiment a number of times.

To quickly do an experiment with more number of moves, instead of using just 1 coin or cube, we will roll 10 cubes at the same time in the next experiments. One move of 10 cubes is equal to 10 moves of 1 cube.

Experiment 2

A histogram of many moves

All the students in the class should be divided into pairs of 2 students each. You shall do this experiment in pairs.

For this experiment you should put dots on 3 sides of each of 10 cubes (just like for the second method of experiment 1). Remember, its heads if the dot is on the top and tails otherwise.

For each move in this experiment we will throw 10 cubes at the same time. There will be 100 such moves in the whole experiment. Count the number of cubes with heads in each move. Make a histogram of the number of heads in each move.

Method of making a histogram

Take **a graph paper** from your kit copy. Make a thick pencil line on one of its horizontal lines (figure 3(a)).

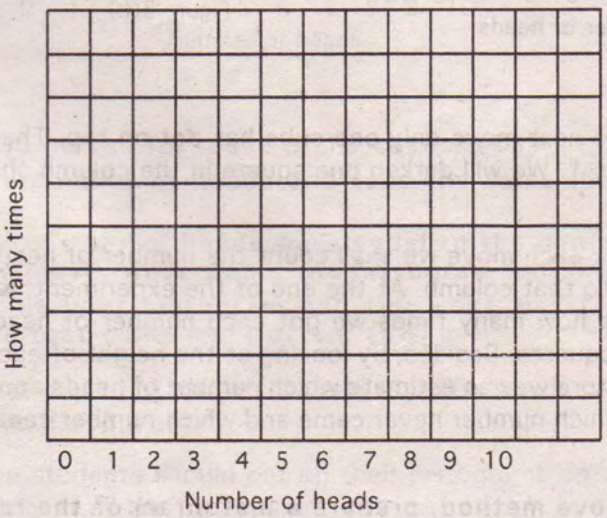


Figure 3(a)

Each time the cubes are thrown, we will see how many cubes have dots on the top. This is the number of heads in that move.

If all the cubes are blank on top, then the number of heads will be zero.

If it happens that 10 out of 10 cubes have dots on top, then the number of heads is 10.

In this way, it is possible to get any number of heads from 0 to 10.

Write the numbers 0,1,2,....9,10 on the graph paper, as shown in figure 3(a). These are the numbers of heads. Now we shall note down on this paper how many times we get each number of heads.

For example, suppose in the first move, 6 cubes fell with dots on the top. The number of heads is 6. Therefore we will use a pencil to darken one square in the column meant for 6 heads (figure 3 (b)).

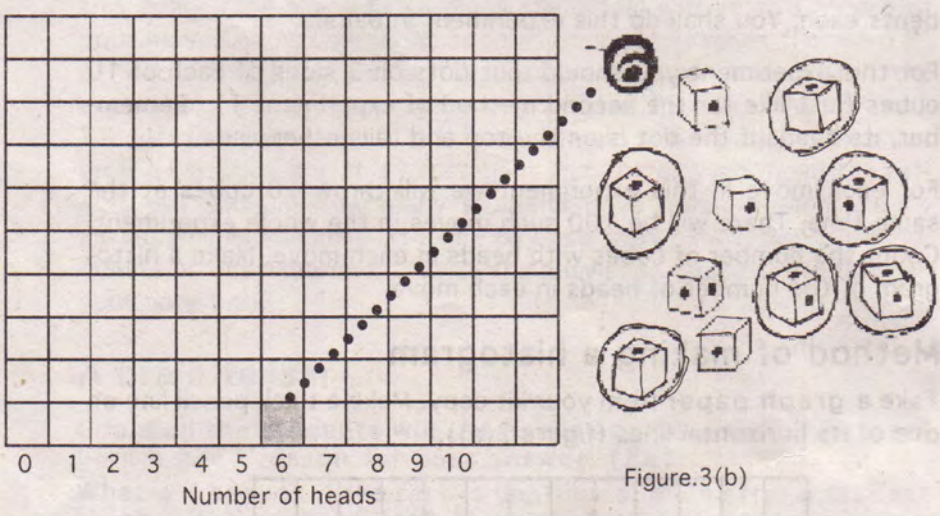


Figure.3(b)

Suppose in the next move only one cube has dot on top. The number of heads is 1. We will darken one square in the column showing one head.

In this way, for each move we shall count the number of heads and add a square to that column. At the end of the experiment, we can easily find out how many times we got each number of heads, by counting the squares. Besides, by looking at the height of each column in the picture we can estimate which number of heads appeared most often, which number never came and which number came less often.

Using the above method, prepare a histogram of the results

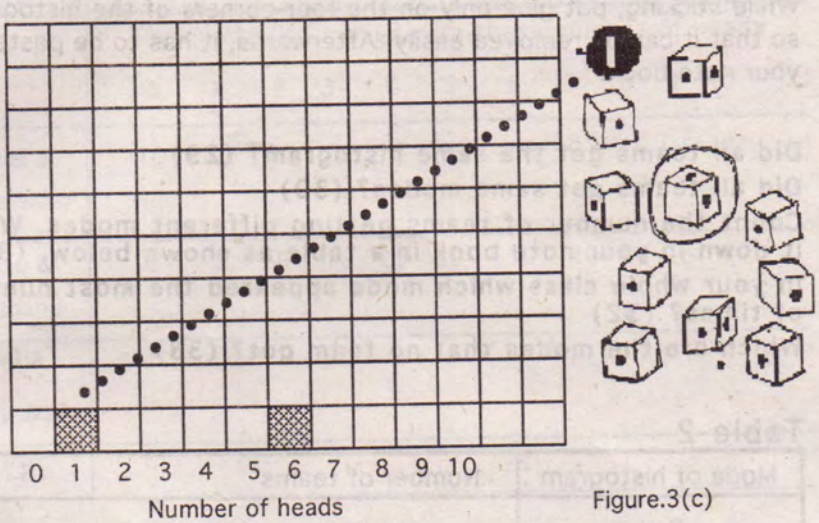


Figure.3(c)

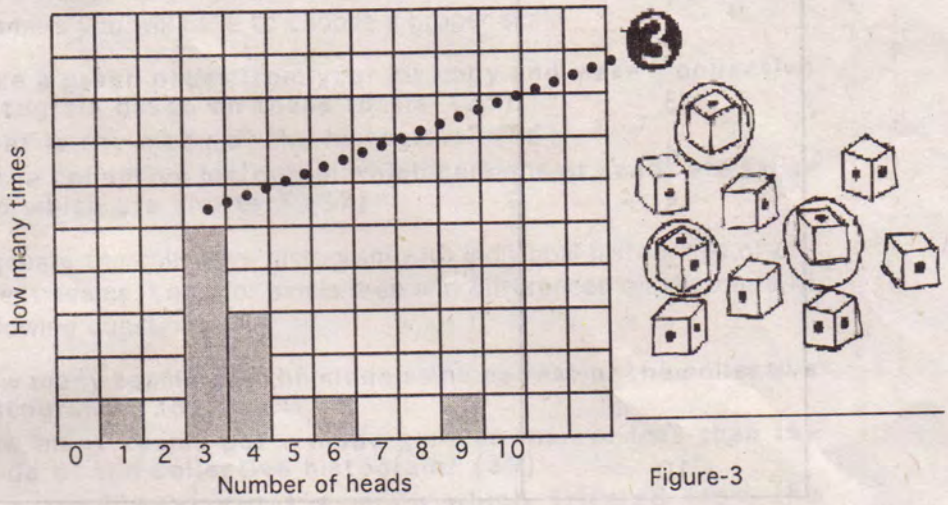


Figure-3

of 100 moves for a set of 10 cubes. (25)

Is the number of heads and tails the same in every move? (26)

If the number of heads were equal to the number of tails in every move, what would the histogram look like? (27)

Look at your histogram and tell what was the number of heads that came most often. (28)

This number of heads, which comes most often, is called the **mode of the histogram.**

All the students should put up their histograms on the wall. Look carefully at all the histograms.

While sticking, put glue only on the four corners of the histogram, so that it can be removed easily. Afterwards, it has to be pasted in your note book.

Did all teams get the same histogram? (29)

Did all teams get same modes? (30)

Count the number of teams getting different modes. Write it down in your note book in a table as shown below. (31)

In your whole class which mode appeared the most number of times? (32)

Which are the modes that no team got? (33)

Table-2

Mode of histogram	Number of teams
0	
1	
2	
3	
4	
5	
..	
..	
..	
..	
10	

Experiment 3

Collective histogram

In this experiment we shall make a collective histogram by putting together experiment 2 data of all the teams.

Make a table in your note book just like Table 3. Look carefully at all the histograms one by one.

In this table, show in how many moves each number of heads was got by different teams. In the last row the totals for all the teams shall be filled. (34)

Some numbers have been filled in Table 3 as examples. Your numbers will be different from these.

Table 3

Name of	The number of moves with these number of heads										
	0	1	2	3	4	5	6	7	8	9	10
Ramlal & Balkrishna	10	0	10	5	15	20	10	0	25	5	0
Balulal & Satanand	0	5	5	10	25	20	10	10	10	5	0
Guddi & Shashikala											
Total											

Look at the totals in the last row of Table 3. Bear in mind that these sums will be quite large. Therefore to make a histogram of these numbers you will have to choose a proper scale.

Take a graph paper from your kit copy and make a collective histogram based on these totals. (35)

What is the mode of the histogram? (36)

In the collective histogram which columns of heads are taller and which are shorter? (37)

Compare the collective histogram with individual histograms of different teams. Look for similarities and differences on the basis of following questions.

How many teams got the mode same as that of the collective histogram? (38)

How many teams got a mode greater than or less than the mode of the collective histogram? (39)

How many teams got a mode which differed from the collective mode by more than 1? (40)

Are the taller columns in most of the team histograms the same as in the collective histogram? (41)

Two main questions had arisen after the first game of heads and tails. Now, after playing so many moves and making the histograms, what opinion have you reached regarding these questions? Discuss it with your teacher and write it down.

If you make another one hundred moves, would your mode shift to :

the right (between 7 and 10), or

to the left (between 0 and 3) of the histogram?

Explain your answer. (43)

If after doing this experiment you are told to make another 100 or 200 moves in the same way, can you predict beforehand what would be the mode of these moves? Or between which numbers would it lie? Write down an explanation. (44)

If after doing so many experiments, having so much experience, and making so many hundreds of moves, you are now asked how many heads will you get in the next move, can you make the right prediction? Explain your answer. (45)

Compare your answers to questions (44) and (45). While you can make a prediction about 100-200 moves but not about your next move. Why is that? (46)

Averages from a histogram

You learned how to find averages in class 6 in the chapter, "Variation and Approximation". Now you will learn a way of finding the average from a histogram. To get the average from a histogram, you have to multiply the number of heads by the number of squares in each column. Add up these products from all the columns. Then divide this sum by the total number of moves (100) to get the average number of heads in each move. For example, the histogram of Ramlal and Balkrishna is shown in Figure 4. The way to get the average of this histogram is given below :

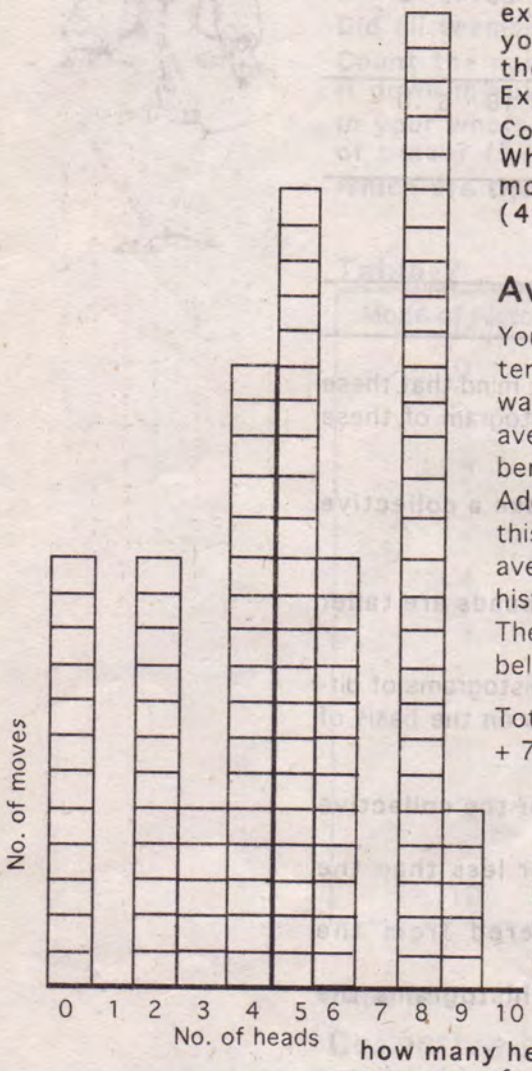


Figure-4

Total = 0x10 + 1x0 + 2x10 + 3x5 + 4x15 + 5x20 + 6x10 + 7x0 + 8x25 + 9x5 + 10x0 = 500.

Average number of heads in each move = 500/100 = 5

Now find the average of your histogram. (47)

Find the average of the collective histogram. (48)

(Bear in mind that the total number of moves for the collective histogram is not 100.)

If 10 cubes were thrown 1000 times, about how many heads would you expect? Base your answer on the average of the collective histogram. (49)

What is Probability

Find out the average number of heads per cube by dividing the average of the histogram by the number of cubes used in the experiment. (50)

This average number of heads per cube is called the **Probability** of getting heads when one cube is rolled. In other words, probability tells us that if we roll one cube, with what surety we can predict a

head - completely, half, or one quarter, or some other measure.

What is chance?

In experiment 2 you rolled 10 cubes 100 times.

100 moves of 10 cubes are equal to how many moves of one cube? (51)

Take a look at the collective table of Experiment 3 and tell how many moves of one cube are equal to the sum total of moves made by the whole class? (52)

From so many moves made in the experiment by your entire class, approximately how many times did heads come up:

Every time?
Three fourth of the times?
half the times?
one fourth of the times?
very few times? (53)

You can guess how many times you will get heads, if you roll a cube many times. You have also found out the probability of getting heads if you roll a cube once. But even so, can you say for sure whether you will get heads or tails when you roll one cube or toss one coin?

You can get anything - heads or tails - when you roll a cube once or toss a coin once. But whatever you get is a matter of sheer chance.

Even though we know the probability, because of chance, we can not predict the result of any single move. However, based on the probability, we can predict approximate total number of heads we would get if we perform many moves.

In the beginning of this chapter an example was given of a particular kind of brinjal plants, which bore an average of 8 brinjals. In other words one plant had a high probability of bearing 8 brinjals.

In spite of this knowledge why can you not say for sure whether one plant will bear more or less than 8 brinjals? (54)

Similarly, knowing that the Allahabad - Itarsi Passenger has a very high probability of coming to Bankhedi late, why you can not say for sure whether this train will be late or on time tomorrow? (55)

When playing a game of*changa*, why you can not predict every time with certainty what number will come in the next move? (56)

An exercise

A farmer had some paddy seeds which were several years old. He decided to test these seeds before planting them. He separately

planted different numbers of seeds and found out what percent of them sprouted. He did a total of 5 experiments and their results are shown in table 4.

Table 4

Expe. No.	No./quantity of planted seeds	No./quantity of sprouted seeds	No./quantity of percentage sprouted seeds
1.	1	0	
2.	10	10	
3.	150	60	
4.	1000	650	
5.	1/2 kg	approximately 2/3	

Fill in the percentage of sprouted seeds in each experiment.

Based on them, tell which experiment the farmer should trust the most to find out the percentage of seed germination? Explain your answer. (57)

What could you say about germination of seeds on the basis of experiment 1 and 2 done by the farmer? (58)

Why is there such a big difference between the results of the first two experiments (1 and 2) and the last two experiments (4 and 5)? Can you give a reason for this difference, on the basis of what you have learned in this chapter? (59)

Experiment 4

Repeat experiment 2 with 10 cubes having a dot only on one side.

Each team should make a histogram of the number of heads from 100 moves. (60)

What is the mode of your histogram? (61)

Compare your histogram with other teams. Do all teams have the same mode? (62)

Add the data from all the teams and make a collective table. (63)

Make a collective histogram on the basis of data so obtained. (64)

Find the average number of heads per cube per move from both the collective histogram and your own histogram. (65)

So, what is the probability of getting heads? (66)

If your cubes have no dots on any side, what would be the probability of getting heads? (67)

If there were dots on all the six sides of your cubes, what would be the probability of getting heads? (68)

Practising probability

Take another look at table 4 and tell how much hope should the farmer have about germination of any one seed. (69)

If you try to pick ace of hearts from a pack of cards with your eyes closed, what is the probability of getting it. (70)

Instead, if you are asked to pick a four of diamonds, is the probability in this case any different from the probability of picking out the ace in the first instance? Give reasons for your answer. (71)

What is the probability of getting heads when you toss one coin? (72)

And how much is it for the tails? (73)

Is finding probabilities always so easy?

During monsoon months black clouds were casting shadows. People thought that there would be torrential rains that day. Every person fully expected it to rain. But suddenly a gusty winds started blowing, and the clouds retreated. That day not even a drop of rain fell.

It did not rain, so does that not mean that there was also a small but definite probability of it not raining? (74)

Can we calculate this small probability, of rain not falling, as easily as the probability of heads or tails when a coin is tossed? (75)

Why is it so difficult to calculate the probability of raining? (76)

Everyday there are events all around you for which it is not always possible to calculate the probability. All you can say is that the probability is small or large.

Think of some examples of such events and write them down. (77)

An extra assignment to be done at home (for enthusiasts)

You must have played *changa* with cowries or *chiye*.

In 1000 moves of *chiye* or cowries, find out how many times you get 1, 2, 3, 4, and 8 ? (78)

Based on this data, find out the probabilities of getting 1, 2, 3, 4, and 8. (79)

May be now you will understand why its difficult to get 4 and 8 in a game of *changa*, and why 2 comes often.

Some experiences - yours and ours

Answer the following questions, on the basis of what you have learned in this chapter :

Doctors test for T.B. by looking for the bacteria in sputum, through a microscope. If they do not see any bacteria in the first test, they collect sputum 3 or 4 more times.

Why is this done? (80)

In experiment-1 of Chapter 'Living world through a microscope', you were told to look for living organisms in pond water. You were also told to look at 4 or 5 different drops through the microscope.

Why was it not sufficient to look at just one drop under the microscope in that experiment? Think about it. (81)

Have you ever seen a gram sevak collecting soil specimens to test the soil in a field? They wander around different parts of the field and collect a little soil from each part, and then mix them together thoroughly. After that a small part from this mixture is taken out and sent to a laboratory for testing.

For finding out the properties of the soil in a field, why is it not thought sufficient to collect soil sample from one place only? (82)

NEW WORDS:

chance	laboratory	probability
mode	collective table	histogram
T.B.	lecturer